

2007 MathEdge+ Advanced Level Contest

Please box the answer. No partial credit will be given. Only correct answer counts. No Calculator. 45 mins.

- 1) What is the remainder when 100000000001 is divided by 3?
ANS: 2
SOL: Take the remainder of the sum of all digits divided by 3. The sum is $1+1=2$. The remainder is 2.
- 2) Find the unit digit of $7 \times 9 \times 11 \times \dots \times 2003 \times 2005 \times 2007$.
ANS: 5
SOL: Because this is the product of odd numbers which will contains at least one five. "Any odd number" $\times 5$ will have "5" as the unit digit.
- 3) A book requires **4221** digits to number the pages starting with page 1. How many pages does this book contain?
ANS: 1332
SOL: 9 (1-digit) + 90×2 (2-digits) + 900×3 (3 digits) = 2889; There are $(4221 - 2889) \div 4 = 333$ (4-digits numbers from 1000 - 1332). The last page number would be 1332. Thus, there are total of **1332 pages in the book.**
- 4) How many digits are there when the product of $3^3 \times 4^5 \times 5^{13}$ is written in decimal notation?
ANS: 14
SOL: $3^3 \times 4^5 \times 5^{13} = 3^3 \times 5^3 \times 4^5 \times 5^{10} = 15^3 \times 100^5 = 15^3 \times 10^{10} = 2250 \times 10^{10}$ <= This number should have 14 digits.
- 5) Two prime numbers P and Q have the property that both their sum and their difference are again prime numbers. What are P and Q?
ANS: 2 and 5
- 6) How many 5-digit positive integers have the property that all their digits are odd and the product of the digits is not a multiple of 5?
ANS: 1024
SOL: Odd digits are 1, 3, 5, 7, 9. If want the product of digits not multiple of 5, then can't use "5" in forming the number. Since 4digits (1,3,7,9) can be used as the digit in the 5-digit number, the number of possible of integers are $4^5 = 1024$.
- 7) Let x be a positive integer. If the least common multiple (LCM) of 2002 and x is 30030, how many different possible values of x are there?
ANS: 16
SOL: Factor $2002 = 2 \times 7 \times 11 \times 13$. $30030 \div 2002 = 15$. X would be a multiple of 15 times any combinations of " $2 \times 7 \times 11 \times 13$ ". " $2 \times 7 \times 11 \times 13$ " make $2^4 = 16$ combinations.
- 8) Alisa's phone number is a 7-digit number where each digit is greater than each of the digits to its left. Her phone number doesn't start with 0 or 1. How many different phone numbers can be her phone number?
ANS: 8
SOL: There are 8 possible numbers can be used, namely: 2,3,4,5,6,7,8. To form a 7-non-repeating digit number using 8 numbers, it has ${}_8C_7 = 8$ ways.

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- 9) Elsa takes 1.5 times as long as her brother to eat a pizza. If they share a pizza (each eating at their own rate) they would take 10 minutes. How long (in minutes) would it take Elsa to eat the pizza all by herself?

ANS: 25

SOL: Assume her brother takes x minutes to eat a pizza, then it would take Elsa $\frac{3}{2}x$.

The combined rate = $\frac{1}{x} + \frac{1}{\frac{3}{2}x} = \frac{5}{3x}$; $\frac{5}{3x} \times 10\text{mins} = 1$; $x = \frac{50}{3}$ mins. The time that

takes Elsa to eat a pizza alone = $\frac{3x}{2} = \frac{3}{2} \times \frac{50}{3} = 25\text{mins}$

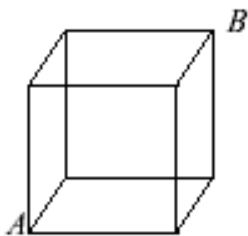
- 10) A and B are riding bicycles around a circular track starting at the opposite side of the circle (i.e., at the two endpoints of a diameter) and at the same time. If both of them ride in the same direction, they will meet after 80 minutes. If both of them ride facing each other, they will meet after 10 minutes. A is riding faster than B at a constant speed of 36km/hr. Assume also that B is riding at a constant speed. What would be B's speed?

ANS: 28

SOL: The distance traveled counter clockwise = the distance traveled clockwise. Let's B be B's speed.

$$(36 - B) \cdot \frac{80 \text{ min } s}{60 \text{ min } s / \text{hr}} = (B + 36) \cdot \frac{10 \text{ min } s}{60 \text{ min } s / \text{hr}} ; B = 28 \text{ km/hr}$$

- 11) How many shortest distances along the edges of the cube are there that connect vertex A with the opposite vertex B?



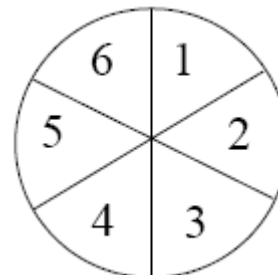
ANS: 6

SOL: To start out from A, there are 3 paths. After that there are only 2 direct paths to B. Thus there are $3 \times 2 = 6$ short path from A to B.

- 12) A pizza is cut into six pie-shaped pieces. John can choose any piece to eat first, but after that, each piece he chooses must have been next to a piece that has already been eaten (to make it easy to get the piece out of the pan). In how many different orders could he eat the six pieces?

ANS: 96

SOL: John can start with any one of the six pieces. After that, he will have only two choices for the next piece, so there are $6 \times 2 = 12$ ways for him to eat the first two pieces,



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namely:

piece 1 followed by piece 2 or piece 1 followed by piece 6
 or piece 2 followed by piece 3 or piece 2 followed by piece 1 or piece 3 followed by
 piece 4 or piece 3 followed by piece 2 or piece 4 followed by piece 5 or piece 4
 followed by piece 3 or piece 5 followed by piece 6 or piece 5 followed by piece 4 or
 piece 6 followed by piece 1 or piece 6 followed by piece 5

For each of these choices, there will be two ways to eat the third piece, then two ways
 to eat the fourth piece, then two ways to eat the fifth piece, and finally only one choice
 for the last piece. We have to multiply by 2 each time we have two choices. So the total
 number of orders in which the pieces could be eaten is

$$6 \times 2 \times 2 \times 2 \times 2 \times 1 = 96.$$

- 13) Amy, Bart and Carol are eating carrot sticks. Amy ate half the number that Bart ate,
 plus one-third the number that Carol ate, plus one. Bart ate half the number that Carol
 ate, plus one-third the number that Amy ate, plus two. Carol ate half the number that
 Amy ate, plus one-third the number that Bart ate, plus three. How many carrot sticks
 did they eat altogether ?

ANS: 36

$$\text{SOL: } A = \frac{B}{2} + \frac{C}{3} + 1; B = \frac{C}{2} + \frac{A}{3} + 2, \text{ and } C = \frac{A}{2} + \frac{B}{3} + 3.$$

$$A + B + C = \frac{A + B + C}{2} + \frac{822}{73} + (1+2+3),$$

which means that

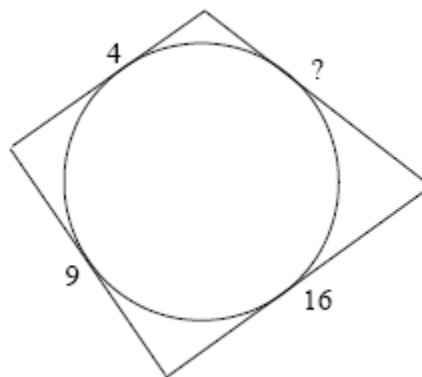
$$(1 - 1/2 - 1/3) A + B + C = 6 \text{ or } 1/6(A + B + C) = 6. \text{ Therefore } A + B + C = 6 \times 6 = 36$$

- 14) A quadrilateral circumscribes a circle. Three of its sides
 have length 4, 9 and 16 cm, as shown. What is the
 length in cm of the fourth side?

ANS: 11

**SOL: The height for the quadrilateral is the diameter (D)
 of the circle. Using area formula for trapezoid:**

$$\frac{(4+16)D}{2} = \frac{(9+?)D}{2}; ? = 20 - 9 = 11.$$



- 15) Len eats a meal at a restaurant and then gets his bill. He
 also has a gift certificate that he could use to reduce his
 bill. He notices that if he uses the gift certificate and then adds on a tip of 20% of the
 reduced bill, the new total is exactly \$1 less than the original bill; however if he adds
 on 25% of the reduced bill instead, the new total is exactly \$1 more than the original
 bill. How much is the gift certificate worth?

ANS: \$9

**SOL: Since adding on 25% of the reduced bill results in a total which is \$2 more than
 adding on just 20% of the reduced bill, that extra 5% must account for exactly the
 additional \$2. That is, \$2 must be exactly 1/20 of the reduced bill, so the reduced bill
 must have been \$40. Adding on 20% would give a total bill of \$40 + \$8 = \$48, which we**

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are told is \$1 less than the original bill, so the original bill must have been \$49.
Therefore the gift certificate must be worth \$49 – \$40 = \$9.

- 16) Garg and Singh played 10 games of badminton together. In each game, whoever scores 15 points is the winner (i.e. the winner need not win by at least two points). It turned out that Garg won more games than Singh did, but Garg got fewer total points than Singh did. What is the largest possible total number of points Garg could have?

ANS: 143

SOL: First look at how many points Singh could get at most. Since Garg won more games than Singh, Singh could have won at most 4 games, in which she scored a total of $15 \times 4 = 60$ points. In the other 6 games Singh could have scored at most 14 points each, so the most number of points she could score altogether is $14 \times 6 + 60 = 84 + 60 = 144$. Since Garg got fewer points than Singh, the most number of points he could score is 143. (This is possible, because Garg gets 15 points in each of the 6 games he wins, for a total of 90 points, and then gets 53 points altogether in the other 4 games, for instance by getting 13 points in three of the games and 14 points in the fourth game.)

- 17) Notice that $\frac{1}{2} - \frac{2}{6} = \frac{1}{6}$. What number A makes the equation $\frac{2}{3} - \frac{3}{A} = \frac{2}{A}$ true?

ANS: $\frac{15}{2}$

SOL: $\frac{2A-9}{3A} = \frac{6}{3A} \rightarrow 2A-9=6 \rightarrow A = \frac{15}{2}$

- 18) How many different three-digit numbers divisible by 25 can be made with the digits 0, 3, 5, 7 if the digits can be repeated?

ANS: 9

SOL: Total 9: 300, 500, 700, 350, 550, 750, 325, 525, 725

- 19) Who is right?

Andy: "The largest number possible has 0 as the last digit."

Bill: "The largest number is googolplex."

Clark: "The largest number possible has 1000 digits."

Danny: "The largest number possible has all 9's."

Elvis: "The largest number possible cannot be written."

ANS: Elvis

SOL: The largest number has infinite number of digits which will never end.

- 20) There are four congruent squares. In each of them the midpoints of the sides are indicated and some regions with areas S_1 , S_2 , S_3 and S_4 are shaded. Which expression below is true?

A) $S_3 < S_4 < S_1 = S_2$

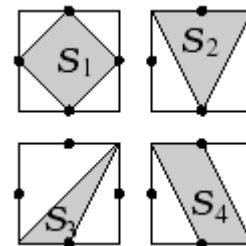
B) $S_3 < S_1 = S_2 = S_4$

C) $S_3 < S_1 = S_4 < S_2$

D) $S_3 < S_4 < S_1 < S_2$

E) $S_4 < S_3 < S_1 < S_2$

ANS: B

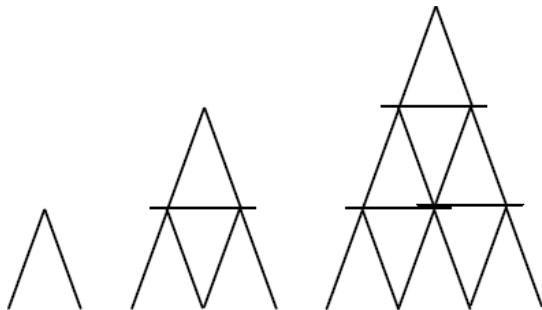


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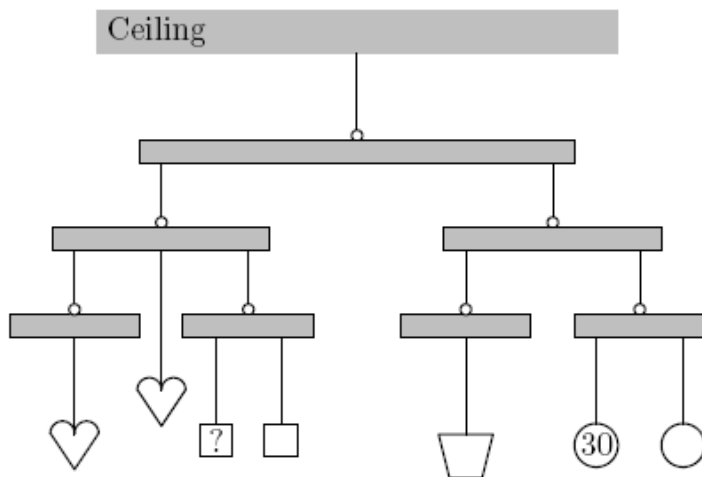
SOL: $S_1=S_2=S_4 > S_3$

- 21) Johnny builds a house made out of cards. In the picture, one-floor, two-floor, and three-floor such houses are shown. How many cards does Johnny need to build 4-floor house?



ANS:26

- 22) The weights in the figure are in balance. The same shapes have the same weight. The weight of each circular shape is 30 ounces. What is the weight of the square shape indicated with the question mark?



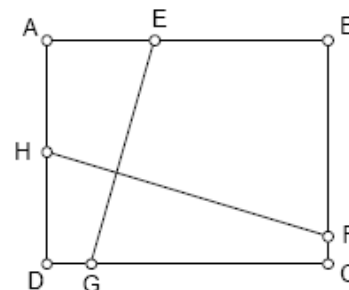
ANS: 20

SOL: $1H = 2B$; $3H = 4C = 4 \times 30 = 120 \Rightarrow H = 40$; $B = 40 \div 2 = 20$

- 23) In the figure, ABCD is a rectangle. E, F, G, H are points on AB, BC, CD, DA respectively with $EG = 3$, $FH = 4$. Find the largest possible perimeter of the rectangle.

ANS: 14

SOL: To have the largest perimeter, $EG = AD$ and $FH = CD$. Thus, the perimeter = $(3 + 4) \times 2 = 14$.



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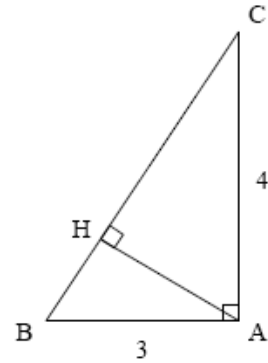
- 24) Let ABC be the right triangle with AB =3, BC =5, AC =4.
Find the length of the altitude AH from A to BC.

ANS: $\frac{12}{5}$

SOL: Using Pythagorean triple (3,4,5): BC = 5. Area =

$$\frac{(3 \times 4)}{2} = \frac{(5 \times AH)}{2} \Rightarrow$$

Thus, $AH = \frac{12}{5}$



- 25) Two of the numbers located on the two circles (see the picture) are represented by letters A and B. The sum of the numbers on each circle is equal to 55. What number is represented by letter A?

ANS: 10

SOL: Sum of lower circle = $47+B = 55 \rightarrow B = 8$; Sum of upper circle = $37 +A+B = 55 \rightarrow A+B = 18$. Since $B = 8$, then $A = 18 - 8 = 10$.

