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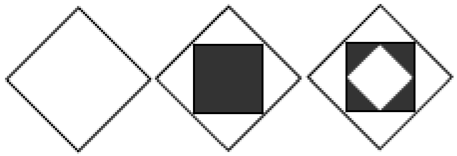
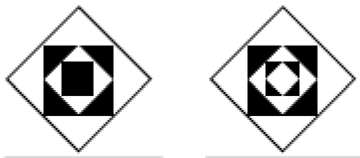
MathEdge Advance Level Contest Problems

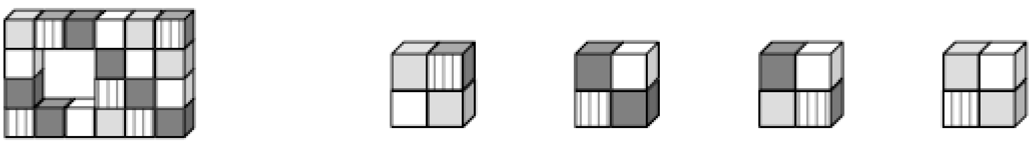
1) (1234567 × 1234567) – (1234566 × 1234566) = ?
ANS: 2469133
SOL: $1234567^2 - 1234566^2$ = (1234567 + 1234566) (1234567 - 1234566) = (1234567 + 1234566) x 1 = 2469133



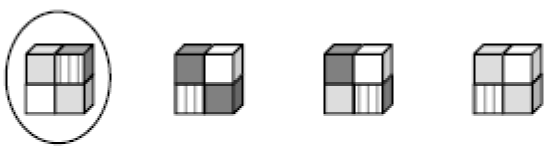
2) Draw the next two pictures: ________ _______
ANS:



3) Circle the shape at the right which fits into the open space of the graph given below:



ANS:



4) Green and blue marbles are placed in the following way: one green, one blue, two green, two blue, three green, three blue, and so on. How many of the first 100 marbles are green?

ANS: b) 55 green marbles
SOL: a) First, we want to find the largest number of marbles we can place so that there are an equal number of each colour. Since the most we can have of either colour (if the two have equal amounts) is 50, we are looking for the largest sum of 1 + 2 + 3 + ... that is less than 50. We can see that 1 + 2 + 3 + ... + 8 + 9 = 45 . That means that after placing nine green and nine blue marbles, we have 45 + 45 = 90 marbles placed. We then place 10 green marbles, and that takes us up to 100 marbles. Therefore, the 100th marble placed is green.
b) We know from part a) that of the first 90 marbles, 45 are green. We also know that the last 10 are green. This gives a total of 55 green marbles.

5) The numbers in the sequence 3, 8, 13, 18, and so on, increase by 5's. The numbers in the sequence 5, 9, 13, 17, and so on, increase by 4's. The number 13 is in both sequences. What is the next number that appears in both sequences?
ANS: 33
SOL: Since one sequence increases by four each time and the other increases by five each time, the two will match up every twenty (four times five). Therefore, if they match up at thirteen, they will again match up at 13 + 20 = 33.

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6) For 15 weeks during the summer, Lesley had a job mowing her neighbour's lawn. When she accepted the job, her neighbour offered to pay a total of \$300 for the 15 weeks or 1¢ for the first week, 2¢ for the second week, 4¢ for the third week, and so on, doubling her pay each week. Would you choose \$300 or 1¢ + 2¢ + 4¢ +…? Explain please.
ANS: 1¢ + 2¢ + 4¢ +… because it becomes \$327.67 which is greater than \$300.
SOL: $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + \ldots + 2^{14} = 2^{15} - 1 = 32768 - 1 = 32767$¢ = \$327.67

7) What is the remainder when 100000000001 is divided by 3?
ANS: 2
SOL: Take the remainder of the sum of all digits divided by 3. The sum is 1+1= 2. The remainder is 2.

8) Find the unit digit of 7 x 9 x 11 x … x 1999 x 2001 x 2003.
ANS: 5
SOL: Because this is the product of odd numbers which will contains at least one five. Odd x 5 will have "5" as the unit digit.

9) Find a prime number p such that 23p + 1 is also prime.
ANS: 2
SOL: All primes are odd except 2. For "23p + 1" to be odd, p has to be even. P = 2 is the only choice.

10) Michael and Fanny each thought of a number. It is known that Michael's number is larger than Fanny's. If the sum of the two numbers is 2002 and their difference is 1001, what is the number Fanny thought of?
ANS: 500.5
SOL: M > F; M + F = 2002 and M – F = 1001. F = (2002 – 1001) ÷ 2 = 500.5

11) $398^2 = 158404$; $3998^2 = 15984004$; $39998^2 = 1599840004$; and $399998^2 = 159998400004$. What is the sum of the digits of **$3{,}999{,}999{,}998^2$** ?
ANS: 85
SOL: According to the pattern in the problem, each time when a 9 is added in between 3 and 8 to the number, the square of the number is added a '9' between 5 and 8, and a '0' is added between 84 and 4. For example, $3998^2 = 15{,}984{,}004$, there are one '9' and two '0's, $39998^2 = 1{,}599{,}840{,}004$, there are two '9's and three '0's.
So 3,999,999,9982 = 15,999,999,984,000,000,004
1 + 5 + (9 × 7) + 8 + 4 + 4 = 85

12) Randomly select two different numbers from this set: **{ -5 , -2 , 4 , 8 }** .
The probability that the **product** of the two selected number is **positive** is
ANS: 1/3
SOL: The product to be positive, both numbers picked have to be either positive or negative. To pick positive numbers, the probability = ½ x 1/3. It has the same probability to pick negative numbers. Thus, the total probability = ½ x 1/3 x 2 = 1/3.

13) M and N are Whole Numbers. 75M is a perfect square. 75N is a perfect cube. The smallest possible value of **M + N** is
ANS: 48
SOL: $75 = 5^2 \times 3$; To make it a perfect square, we multiple it by 3 to become $5^2 \times 3^2$ which means M=3. To make it a perfect cube, $N = 5 \times 3^2 = 45$. Thus M + N = 3 + 45 = 48.

MathEdge Advance Level Contest Problems

14) $(1+\frac{1}{2})(1-\frac{1}{3})(1+\frac{1}{4})(1-\frac{1}{5})\ldots\ldots(1+\frac{1}{n})(1-\frac{1}{n+1})\ldots\ldots(1+\frac{1}{48})(1-\frac{1}{49})(1+\frac{1}{50})$

What is the product of these 49 factors?

ANS: $\frac{51}{50}$

SOL: The product of every 2 terms equal to 1 (3/2 x 2/3 = 1). The last term $1+\frac{1}{50}=\frac{51}{50}$

15) A 27-year old mother has a 5-year old daughter. In how many years will the mother be three times as old as her daughter?

ANS: 6

SOL: Assume X is the number of years later. 27 + X = (5+X) 3 => 27 + X = 15 + 3X; 2X = 12; X = 6.

16) From 1:08 PM to 5:46 PM, the hour hand of a clock moves through an angle of

ANS: 139^{o}

SOL: Every hour, the hour hand moves 240°÷ 12 = 30°. From 1:08 PM to 5:46 PM, it has moved 4:38 which is 120°(4 x 30°) + 38/60 x 30°= 120°+ 19°= 139°.

17) A square piece of paper is folded in half twice: from top to bottom, then from top to bottom again. If the perimeter of the final rectangle is 35 cm, what was the perimeter of the original square?

ANS: 56

SOL: Let X be the side of the square. After 2 folded in half, one side is shorten to $\frac{X}{4}$.

(X + $\frac{X}{4}$**) x 2 = 35cm; X = 14cm; The perimeter of the original square = 4 X = 4 x 14cm = 56cm.**

18) The natural numbers are written in sequence, without spaces: **1234567891011121314151617.... and so on.** What is the **300th digit** in this sequence?

ANS: 6

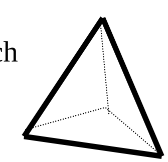
SOL: 300 = 9 x 1 + 90 x 2 + 37 x 3; Thus, the 300th digit is the 37th's 3-digit number from 100 which is 136. Thus the 300th digit is "6".

19) If $x - y = A$ and $xy = B$, then what is $(x + y)^2$ in terms of A and B?

ANS: $A^2 + 4B$

SOL: $(x + y)^2 = (x - y)^2 + 4xy = A^2 + 4B$

20) A fair tetrahedral die has four faces and four vertices. Each vertex is numbered and each vertex is equally likely to "land up". You have two such dice.



On die #1, the vertices are labeled: 1, 2, 3, and 4.
On die #2, the vertices are labeled: 2, 3, 4, and 5.

When these two dice are rolled, what is the probability that the sum of the two "up" vertices is 7?

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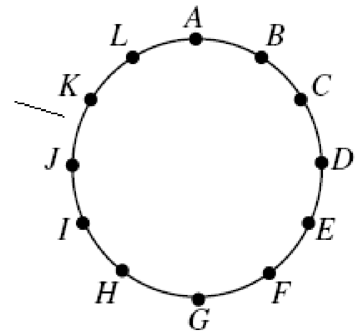
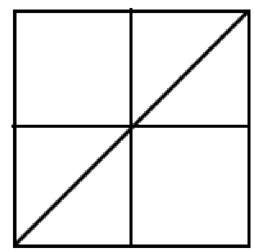
MathEdge Advance Level Contest Problems

ANS: 3/16
SOL: The total different combinations of rolling two dices = 4 × 4 = 16.
To get a sum is equal to 7, the combination of two dices are (2, 5), (3, 4), (4,3),
The probability that the sum of the two "up" vertices is 7 is = 3/16.



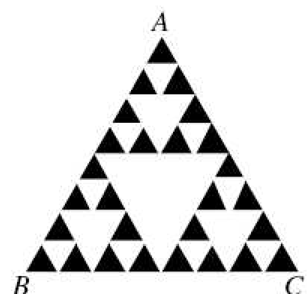
21) Twelve balloons are arranged in a circle as shown. Counting clockwise, every third balloon is popped. C is the first one popped. This continues around the circle until two unpopped balloons remain. What are the the last two remaining balloons?

ANS: E and J
SOL: The first ten balloons are popped in the following order: C, F, I, L, D, H, A, G, B, and K. The remaining two balloons are E and J.



22) In the game "TRISQUARE", three points are awarded for each triangle found, and four points for each square. What is the highest number of points that can be achieved for the given diagram?

ANS: 38
SOL: In the diagram, there are four small and two large triangles, for a total of 18 points. As well, there are four small and one large square, for a total of 20 points. Altogether, 38 points can be achieved.

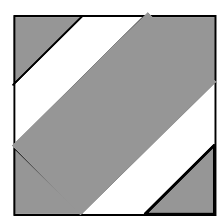


"Sierpinski Gasket"

23) In the diagram, all triangles are equilateral. What fraction of Δ ABC is coloured black?

ANS: 27/64
SOL: In the diagram, there are 27 black triangles. If the entire diagram was divided into the smallest size equilateral triangles, there would be 8 + 2(7) +2 (6) + 2 (5) + 2 (4) + 2 (3) + 2 (2) + 2 (1) = 64 (counting by rows). Thus, 27 / 64 of Δ ABC is coloured black

24) Each side of this square is trisected. What fraction of the square is shaded?



ANS: 2/3
SOL: assume each side of the square is 3 then divide the square into two equal portions via the diagonal that is in the middle shaded portion. Let's solve the one half of the square. The triangle with the shaded and un-shaded area is ½ (2 × 2) = 2. The small triangle is ½ × (1 × 1) = ½. The big triangle on the bottom is ½ × (3 × 3) = 9/2. So 9/2 − 2 + ½ = 6/2 = 3.
3 ÷ 9/2 = 2/3.