IMAS Sample Question
Lower Level (G6 and lower) Round 2

1. （Combinatoric） The regulations imposed that each member of the national men's basketball league can choose his squad number from 0 to 55. However, if you choose a two-digit number, the numeric value of each digit cannot exceed 5. And the number could not be changes once confirmed. How many different combinations of number available for the team to choose ?

(A) 34 (B) 35 (C) 40 (D) 55 (E) 56

Ans：C.

Number can are available are：0～9、10～15、20～25、30～35、40～45、50～55, 40 in total.

2. （Combinatoric） In figure 1, A large grid is composed by 81 small squares of 1cm^2 each . B and C are two points on the grid. A is a point on the grid so that the area of the triangle △ABC is 3 cm^2. How many such different points A are possible?

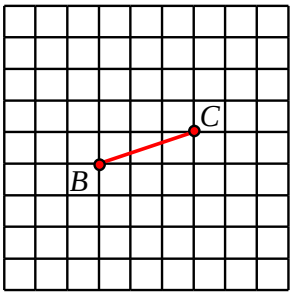


Figure1

(A) 5 (B) 6 (C) 8 (D) 10 (E) 12

Ans 1：C.

Starting from the most upper line, and denote them by 1, 2, …, 10 horizontal lines, and count the points that satisfied the requirement. We got: No point available on line 1, and only one point A_7 on line 2. (as in figure 2). Similarly, we got points A_2, A_1, A_6 that satisfy the requirement. By symmetry, points A_5, A_4, A_3, A_8 also are answer. Hence option C is the answer.

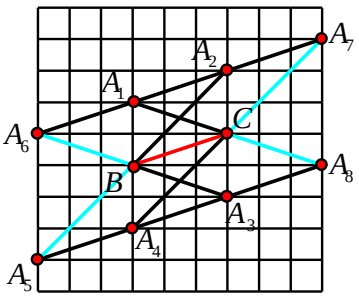


Figure 2

Ans 2：C.

In figure 3, point A_1 obviously is the answer. As

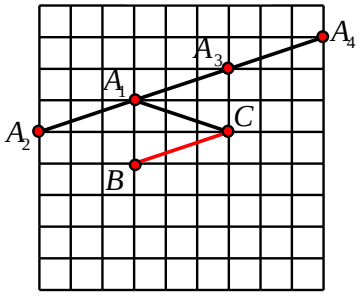


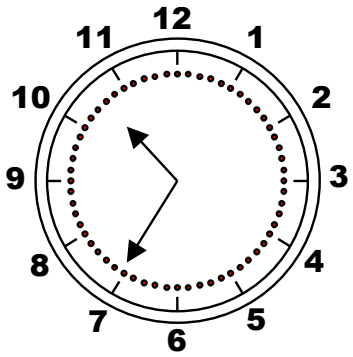
Figure 3

the side BC of $\triangle ABC$ is fixed, points above the line BC that satisfy the requirement will pass through point A_1 and parallel to line BC. From the figure, points A_2, A_3, A_4 all satisfy the requirement. By symmetry, there are 4 such points under the line BC. Hence option C is the answer.

3. （Geometry）An electronic **clock** was installed on the clock tower of a train station. There is a small light on every minute mark along the circumference of the clock. At 9:35:20 p.m., how many lights are on the smaller arc between the hour hand and the minute hand?
(A) 12 (B) 15 (C) 17 (D) 20 (E) 24

Ans：A.

At 9:30:00 pm, the angle subtended by the hour hand and the minute hand is 105º. There are 17 lights within this arc of circle. Five minutes and 20 seconds later, the minute hand pass through 5 lights, and the hour hand did not pass through any light. Hence there are $17-5=12$ lights.



4. （Arithmetic）Jia Xian Triangle（Or Pascal Triangle）:

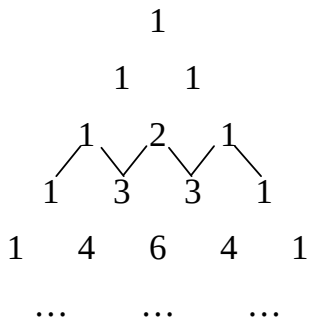
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
… … …

Starting from the third line, the numbers on the two ends of each line are 1. And the value of the number within the interval is the sum of the two numbers just above it at the previous line. What is the sum of all numbers from the first to the tenth line?

Answer：_____

Ans：1023.

Observe the sum of each line. The sum of the first line is 1, the sum for the second line is 2, the third is 4, and the fourth is 8. Hence we can conjecture that starting from line 2, sum of number in each line is twice the sum of number of the above line. This can be checked easily, by the following diagram, each number in the upper line is counted twice in order to obtain the line below.



Hence the sum of number from line 1 to line 10 is: $1+2+2^2+...+2^9=1023$。

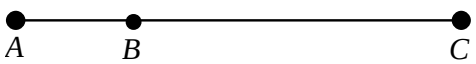
5. （Number Theory）In a game, a number of rectangular bolcks mesaure 2×3×5 is stacked into three piles, each with different number of blocks. The first pile is stacked up with base 3×5. The second pile is stacked with base 2×5. The third pile is stacked with base 2×3. Finally the three piles of blocks are of the same height. How many rectangular blocks are there in total for the three piles.

Answer：_____

Ans：31.

The least height of the pile is 30, the LCM of 2, 3 and 5. Then the number of the blocks in each pile is at least $\frac{30}{2}$, $\frac{30}{3}$, $\frac{30}{5}$. The sum is $\frac{30}{2}+\frac{30}{3}+\frac{30}{5}=31$。

6. （Arithmetic）As in the figure, Tom and Mary are departing from city A and city B respectively and are going to City C. Tom meets Mary at the distance 2500m away from city C. If Mary starts her journey 10 minutes earlier, then Tom will meet Mary at the distance 1000m away from city C. Given that the speed of Mary is 60m per minute. What is the speed of Tom?



Answer：_____m

Ans 1：100.

Mary starts 10 minutes earlier, then the difference in distance of them will increase 10×60 = 600 m, and Tom needed to do 2500 - 1000=1500 m more to

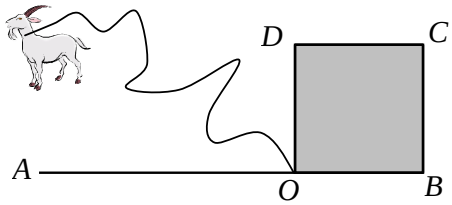
meet Mary. That is, when Tom travelled 1500 m, Mary traveled 1500 - 600 = 900 m. Hence the ratio of speed of Tom and Mary is 1500 : 900=5 : 3. Hence the speed of Tom is 60÷3×5= 100 m per minute。

Ans 2：100.

Assume that the speed of Tom is v m per minute。

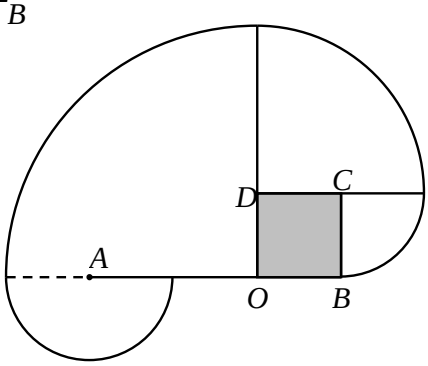
$\frac{600}{v-60}=\frac{1500}{v}$, that is $600v=1500(v-60)$ ，$900v$ = 90000. Hence v = 100.

7. （Geometry）A shepherd tied a sheep with a 6-meter-long rope at point O on a grassland, where ABCDO is a high fence. AO is 4 meters in length and OBCD is a square of side 2 meters. What is the largest area of grass that the sheep can access in m^2 ?



Ans：16π m^2.

The largest area of grass that is available is shown in the diagram：



It is formed by a half circle of radius 2 m and three “one-quarter” circles with radii 2 m, 4 m, and 6 m respectively. The area is

$\frac{1}{2}\times\pi\times2^2+\frac{1}{4}\times\pi\times2^2+\frac{1}{4}\times\pi\times4^2+\frac{1}{4}\times\pi\times6^2$

$=16\pi$ （m^2）。

8. （Combinatoric）The following is operated on a given non-zero natural number. If the number is even, then the number is divided by 2. If the number is odd, then 1 is added to the number. The above operations will continue until the resulting number 1. How many numbers are there to end with number 1 under exactly 9 operations ?

Ans：34.

1 number go through 1 operation ： 2 ；

1 number go through 2 operation ： 4 ；

2 numbers go through 3 operation ： 3 , 8 ；

3 numbers go through 4 operation ： 6 , 7 , 16 ；

5 numbers go through 5 operation : 5，12，14，15，32；

8 numbers go through 6 operation : 10，11，24，13，28，30，31，64；

13 numbers go through 7 operation : 9，20，22，23，48，26，27，56，29，60，62，63，128；

21 numbers go through 8 operation : 18，19，40，21，44，46，47，96，25，52，54，55，112，58，59，120，61，124，126，127，256；

34 numbers go through 9 operation : 17，36，38，39，80，42，43，88，45，92，94，95，192，50，51，104，53，108，110，111，224，57，116，118，119，240，122，123，248，125，252，254，255，512。

Note：The number after n operation is the n-th term of a Fibonacci sequence.