IMAS Sample Question
Upper Level (G9 and lower) Round 1

1. （Algebra）Two different liquid A and B are mixed according to the ratio of x:y (in weight) to make a new drink. The cost of liquid A is $5 per 500g and for B is $4 per 500g. Now, the cost of liquid A increases by 10% and the cost of liquid B decreases by 10%, while the cost of the new drink remains unchanged after the changes. What is the value of x: y ?

   (A) 1：2 (B) 2：3 (C) 2：5 (D) 3：4 (E) 4：5

Ans：E.

By $5x+4y=5.5x+3.6y$ ，$5x=4y$ ，$x:y=4:5$。

2. （Algebra）There are two paper strips. The longer one is 23cm in length, whilst the shorter one is 15 cm. A certain equal length of paper was cut off from both strips, with a result that the remaining length of the longer one is at least two times the shorter one. What is the length of the paper being cut.?

   (A) 6 cm (B) 7 cm (C) 8 cm (D) 9 cm (E) 10 cm

Ans：B.

Let the length of paper being cut is x cm, then

$$23-x \geq 2(15-x) \ ,$$

And $x \geq 7$. Hence the length of paper cut is at least 7 cm.

3. （Combinatoric）A certain number of ping pong balls were put into 10 boxes, and the numbers of balls in each box are different. However, the number of balls in each box could not be less than 11, could not be equal to 13 and could not be the multiple of 5. What is the minimum number of ping pong balls needed?

   (A) 150 (B) 155 (C) 162 (D) 173 (E) 186

Ans：D.

At lest $11+12+14+16+17+18+19+21+22+23=173$ needed.

4. （Number Theory） Along one side of a 3000-metre new road , a certain number of lamps-posts are placed. According to the original design, the distance between two lamp-posts is 50 metres and the digging work was completed. If the distance between the posts is now changed to 60 metres, how many new holes are needed to be dug?

   (A) 11 (B) 40 (C) 50 (D) 60 (E) 61

Ans：B.

The original number of holes dug is $\frac{3000}{50}+1=61$.

The new number of holes is $\frac{3000}{60}+1=51$. The LCM of 50 and 60 is 300, hence holes that are 300 m apart are maintained, which equal to $\frac{3000}{300}+1=11$. Hence the new work needed is $51-11=40$ holes.

5. （Number Theory） In the following five numbers,

$$1234554321、1234554321^2、123454321、$$

$$123454321^2、123454321\times1234554321,$$

How many multiple of 2009 are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Ans：B.

We know that

$2009=7^2\times41$, $111111=7\times11\times13\times111$, $11111=41\times271$,

$1234554321=11111\times111111$ is a multiple of 41 and 7, but not 7^2

$123454321=11111\times11111$ is not a multiple of 7, hence only 1234554321^2 is the multiple of 2009.

6. （Geometry）Lights are placed at every coordinates of integral values on a Cartesian plane (x, y are positive). When t = 0, only the light at the origin is on. When t = 1, 2, …., the lights will switch on according to the condition that the distant of the switch-on light is at least 5 unit from another switch-on light. How many lights could never be switched on the Cartesian plane?

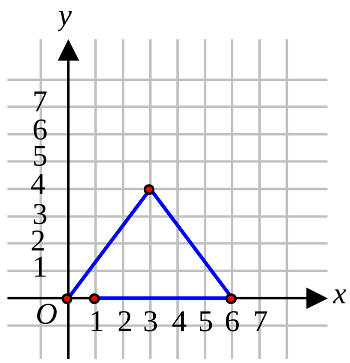
(A) 0 (B) 4 (C) 8 (D) 12 (E) infinite

Ans：A.

All the lights could be switched on.

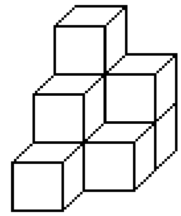
The following is the co-ordinates of sequence of the lights to be switched on （distance of lights being 5）：

(0, 0)→(3, 4)→(6, 0)→(1, 0)。



This means that lights that have a distance of 1 from the origin could be switched on. Hence all lights could be switched on.

7. （Geometry）The figure shown is composed by 9 cubes, with side 1 cm. What is the total surface area of the figure?



Answer：_________ cm^2

Ans：32.

Viewing from top, bottom, front, back, left and right, the surface areas of the figure are 5 cm^2, 5 cm^2, 5 cm^2, 5 cm^2, 6 cm^2, 6 cm^2 , respectively. The total area is 32 cm^2.

8. （Combinatoric）Given the following twelves numbers: 2000, 2001, 2002, … , 2011. How many of them could not be expressed as difference of two perfect squares?

Answer：_________

Ans 1：3.

If a is odd, then

$$a=\left(\frac{a+1}{2}\right)^2-\left(\frac{a-1}{2}\right)^2.$$

If a is a multiple of 4, then

$$a=\left(\frac{a}{4}+1\right)^2-\left(\frac{a}{4}-1\right)^2。$$

If an even number could be expressed as a difference of two squares, then these two numbers must both be odd or even. The difference of squares of two odd (even) numbers must be a multiple of 4. As 2002, 2006, 2010 are not multiple of 4, they could not be expressed as difference of two squares.

Ans 2：3.

A square number will have reminder 0 or 1 when divided by 4. And when the difference of two squares number divided by 4, the reminder could not be 2. Hence 2002, 2006, 2010could not be expressed as difference of two squares numbers.

1. $a^2-b^2=(a-b)(a+b)=2000$, one of the answer is a=501, b=499。
2. $a^2-b^2=(a-b)(a+b)=2001$, one of the answer is a=335, b=332。

3. $a^2 - b^2 = (a-b)(a+b) = 2003$, one of the answer is *a*=1002, *b*=1001。

4. $a^2 - b^2 = (a-b)(a+b) = 2004$, one of the answer is *a*=502, *b*=500。

5. $a^2 - b^2 = (a-b)(a+b) = 2005$, one of the answer is *a*=1003, *b*=1002。

6. $a^2 - b^2 = (a-b)(a+b) = 2007$, one of the answer is *a*=1004, *b*=1003。

7. $a^2 - b^2 = (a-b)(a+b) = 2008$, one of the answer is *a*=503, *b*=501。

8. $a^2 - b^2 = (a-b)(a+b) = 2009$, one of the answer is *a*=1005, *b*=1004。

9. $a^2 - b^2 = (a-b)(a+b) = 2011$, one of the answer is *a*=1006, *b*=1005。