IMAS Sample Question
Upper Level (G9 and lower) Round 2

1. （Number Theory）What is the least positive integer that has 20 factors?
(A) 120 (B) 240 (C) 360 (D) 432 (E) 1536

Ans：B.

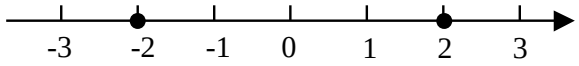
As $20=2\times10=4\times5=2\times2\times5$, the positive integer that has 20 factors is either the multiple of 3 and 2(9 times), which is $3\times2\times2\times2\times2\times2\times2\times2\times2=1536$, or a multiple of 3 (3times) and 2 (4 times), which is $3\times3\times2\times2\times2\times2=432$, or a multiple of 3, 5 and 2 (4 times), which is $3\times5\times2\times2\times2\times2=240$. Hence the least positive integer is 240.

2. （Number Theory）Given that m and n are positive integers, and only one of the following statement is incorrect. Which one of the following statement is incorrect?
(A) $m+n$ is divisible by 3(B) $m+1$ is divisible by n (C) $m+7n$ is prime
(D) $m=2n+5$ (E) mn is even

Ans：A.

If (A) is correct, then $m+7n=(m+n)+6n$ is larger than 3 and is not divisible by3,and is not prime. Hence (C) is incorrect. Option (D) is incorrect, otherwise, by $m=2n+5$ and the correctness of (A), we have：$m+n=(2n+5)+n=3n+5$ being divisible by 3. As $3n+5$ is not divisible by 3, option (D) is incorrect. The above disagree with “one of the options (A), (B), (C), (D), (E) is incorrect”. Hence option (A) is incorrect. In fact, it could be checked with taking $m=17$，$n=6$.

3. (Algebra) Given that the points -2 and 2 divides the number line into three sections. If for arbitrary n different points on the number line, at least three of the points lie in one of the sections. What is the least value of n?



(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Ans：D.

The 3 sections of the number line include all corresponding points, and by Pigeon-Holes theorem, option (D) is correct.

4. （Algebra）A watch is always 3 minutes slow in one hour than a normal watch. If the watch is accurate at the time 4:30 a.m., and later that morning, the hands of the watch show 10:50 a.m. What is the accurate time shown on a normal watch?

Answer：_____

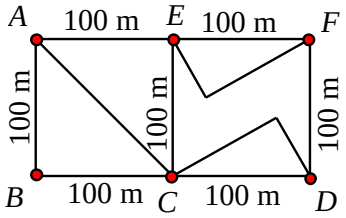
Ans：11:10am.

Let the accurate time be x hour, accordingly

$$\frac{1}{20}\left(x-4\frac{1}{2}\right)=\left(x-10\frac{5}{6}\right)$$

And we have $x=11\frac{1}{6}$（hours）=11:10am.

5. （Combinatoric）As shown in the figure, A represents the post office whilst B,C,D,E and F are 5 households. The distance between each household is shown in the figure. A postman starts from the post office to deliver letters to each household (every household gets a letter). The last stop for the postman is D and he is not required to return to the post office. What is the shortest route in metres covered by the postman?



Answer：_____

Ans：500 m.

$$A \xrightarrow{100} B \xrightarrow{100} C \xrightarrow{100} F \xrightarrow{100} E \xrightarrow{100} D。$$

6. （Geometry）As shown in figure 1, the length of road along the lake-side are AB = 3km, CD = 12 km, and AD = 13km respectively. AB is perpendicular to BC. The shaded area in the figure represent the grassland, the rest is the water. If we take a yacht and start from C, with a speed of 10 km per hour, how much time in hours is needed to reach the other side AD?。

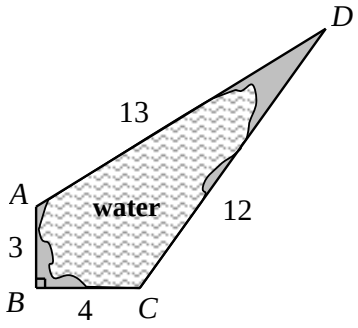


Figure 1

Answer：_____

Ans：$\frac{6}{13}$.

Connect *AC* (as in figure 2), and by Pythagorean theorem, *AC*=5 km. As $5^2+12^2=13^2$, ΔACD is an right angle triangle, $\angle ACD$=90°. To start from C with a speed of 10 km per hour and arriving *AD* with the shortest time, a shortest distance is required, which is the distance from C to *AD*. This is the height of the right angle triangle ACD.

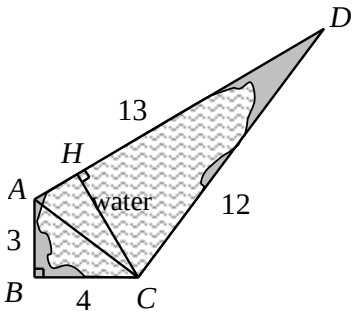


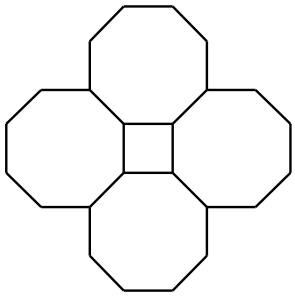
Figure 2

$$\text{Height CH } = \frac{AC \times CD}{AD} = \frac{5 \times 12}{13} = \frac{60}{13}\text{km。}$$

Hence the shortest time required is

$$\frac{60}{13} \div 10 = \frac{6}{13}\text{ hours.}$$

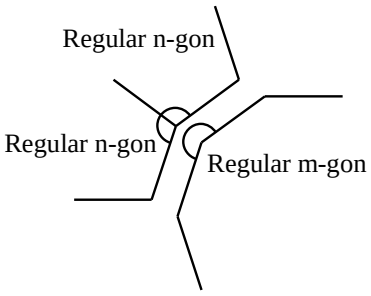
7. （Geometry）A regular m-sided polygon is tessellated by m n-sided polygon (The diagram show the case when m=4 and n=8). When m = 10, what is the value of n?



Ans：

An interior angle of a regular *m-sided* polygon is $\frac{(m-2)\times 180^\circ}{m}$, hence an interior angle of a 10-sided polygon（ *m*=10 ）is $\frac{(10-2)\times 180^\circ}{10} = 144^\circ$.

There are two n-sided regular polygon tessellated

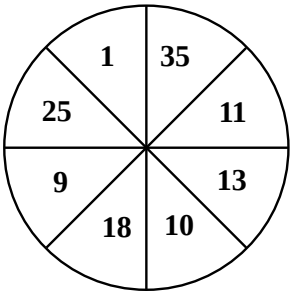


on every vertex of a regular m-side polygon. Hence the angle sum of the interior angle of m-side polygon and two times the interior angle of a n-sided polygon is equal to 360°. And

$$144^0 + 2 \times \frac{n-2}{n} \times 180^\circ = 360^\circ,$$

hence $n = 5$。

8. （Combinatoric）A group of people darts at the board shown (8 sectors). The number shown on sectors of the board represents the number of scores. Everyone darts four times and everyone scores a total of 62. Any two of the players will have at least one score sector different from the other. At most how many people are involved in the game?



Ans：

To solve the problem, we need to know how many of the darts did not hit the target, and the different combinations of at most four of the following numbers (1, 9, 10, 11, 13, 18, 25 and35) to represent the sum 62. The numbers appeared could be repeated. There are only two even numbers here, the scores could be 18 and 18, 10 and 18, 10 and 10, or all numbers are odd. This helps to limit the search area. Hence we have 8 possible outcomes for score 62.

$$\begin{aligned}
\mathbf{62} &= \mathbf{35+25+1+1} \\
&= \mathbf{35+13+13+1} \\
&= \mathbf{35+9+9+9} \\
&= \mathbf{25+25+11+1} \\
&= \mathbf{25+18+18+1} \\
&= \mathbf{25+18+10+9} \\
&= \mathbf{25+13+13+11} \\
&= \mathbf{18+18+13+13}
\end{aligned}$$

There is only one possibility for using 3 darts to strike 62, which is {35, 18, 9}. Since the sum of two largest number appeared on the board is 25+35=60, it is not possible to score 62 by two darts.

Hence at most 9 persons is involved in the game.